Comments on "Data Driven Stability Analysis of Black-box Switched Linear Systems" [Automatica 109 (2019) 108533] ⋆

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Abstract

This item explains a technical problem in the justification of Theorem 10 in Kenanian et al. (2019). The theorem refers to a result in chance-constrained convex programming, but the hypotheses for applying this result are not satisfied since the the optimization problem involved in Theorem 10 is nonconvex. However, under an additional mild assumption on the system, the conclusions of Theorem 10 hold, as shown in this item.

Key words: Learning algorithms, Data-driven verification, Stability, Switched systems, Optimization, Lyapunov methods

1 Problem in the proof of Theorem 10

Theorem 10 in Kenanian et al. (2019) is provided without proof, but with a reference to Theorem 3.3 in Calafiore (2010), suggesting that the former is a particular case of the latter. However, Theorem 3.3 in Calafiore (2010) assumes that the underlying optimization problem is convex, but (8) in Kenanian et al. (2019) is nonconvex (because of the term $\gamma^*(\omega_N)$), so that Theorem 10 in Kenanian et al. (2019) cannot be derived from Theorem 3.3 in Calafiore (2010).

Remark 1. If $\gamma^*(\omega_N)$ in (8) in Kenanian et al. (2019) is replaced by a fixed value (say γ), then the problem becomes convex. In that case, the conclusion that can be drawn from Theorem 3.3 in Calafiore (2010) is that with probability $\beta(\epsilon, N)$ on the sampling of ω_N, either (8) in Kenanian et al. (2019) with $\gamma^*(\omega_N)$ replaced by γ is not feasible, or when it is feasible, then a guarantee similar to (9) in Kenanian et al. (2019) holds. However, there is no way to guarantee in advance that (8) in Kenanian et al. (2019) with $\gamma^*(\omega_N)$ replaced by γ is feasible, so that this modified problem cannot be used for the problem at stake.

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2 A proof based on chance-constrained quasi-convex optimization

It happens that (8) in Kenanian et al. (2019) is very close to being a *quasi-convex* optimization problem. To see this, let $\mathcal{M} = \{A_i : i \in M\} \subseteq \mathbb{R}^{n\times n}$ be as in (2) in Kenanian et al. (2019) and let ω_N be as in (6) in Kenanian et al. (2019), and consider the following optimization problem:

$$\begin{aligned} \min_{P\in\mathcal{S}^n,\gamma} \quad & (\gamma, \|P\|_F) \\ \text{s.t.} \quad & (\mathbf{A_j}x)^\top P\mathbf{A_j}x \le \gamma^{2\ell}x^\top Px, \quad \forall\,(x,\mathbf{j}) \in \omega_N, \\ & I \preceq P \preceq CI,\ \gamma \ge 0, \end{aligned} \tag{1}$$

where $C > 1$ is a parameter, and $\mathbf{A_j}$ is defined as in (7) in Kenanian et al. (2019). The objective of (1) is to minimize γ and then, if there is more than one feasible solution with the optimal γ, find the one such that $\|P\|_F$ (Frobenius norm) is the smallest; hence, this is the same as (8) in Kenanian et al. (2019) with $\eta = 0$ and with $\lambda_{\max}(P)$ replaced by $\|P\|_F$ (the latter is just a technical fix made to ensure that the optimal solution is unique). We have also added a constraint $P \preceq CI$, to ensure that the set of feasible P is compact, so that an optimal solution is always guaranteed to exist.

Problem (1) is a *quasi-linear* optimization problem, as defined in (2) in Berger et al. (2021). Therefore, we may apply Theorem 6 in Berger et al. (2021) to get the following result (which relies on Assumption 4, discussed below):

Theorem 2 (Corollary 12 in Berger et al., 2021)**.** *Consider Problem (1) and let $\mathcal{M}$ satisfy Assumption 4. Let $d = \frac{n(n-1)}{2}$. For any $\epsilon \in (0,1]$ and $N \geq d+1$, the bound (9) in Kenanian et al. (2019) holds, where $P(\omega_N)$ and $\gamma^*_{\omega_N}$ are understood as the optimal solutions of* (1).

Remark 3. Note that d in Theorem 2 is smaller by one unit than the d used in Theorem 10 in Kenanian et al. (2019). Hence, the bound, similar to (9) in Kenanian et al. (2019), obtained by using Theorem 2 is stronger than the bound (9) in Kenanian et al. (2019) since $\beta(\epsilon, N)$ is decreasing with d.

Theorem 2 relies on the following assumption:

Assumption 4 (Assumption 8 in Berger et al., 2021)**.** *$\mathcal{M}$ contains no Barabanov matrix.*

We recall that, by Proposition 9 in Berger et al. (2021), a real square matrix is *Barabanov* if and only if it is diagonalizable and all its eigenvalues have the same modulus. Hence, Assumption 4 can be assumed to hold in most practical situations. We are currently working on alleviating this assumption.

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