First name, last name:

CSCI 5654, Spring 2023: Spot Exam 1

Date: We 3/15/2023

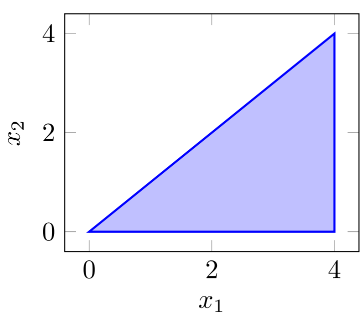
Instructions: For multiple-choice questions, unless said otherwise, there is one and only one correct choice per question.

Question 1

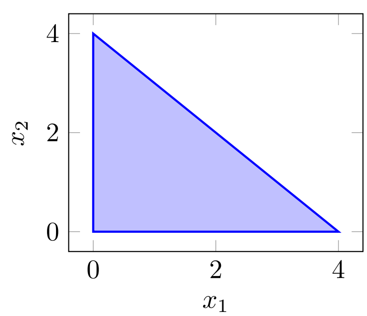
Which plot corresponds to the feasible set of the following Linear Program?

$$\begin{array}{rl} \max & x_1 + 2x_2 \\ \text{s.t.} & 2x_1 \geq x_2 - 4 \\ & 2x_1 \leq x_2 \\ & 0 \leq x_2 \leq 4 \ . \end{array}$$

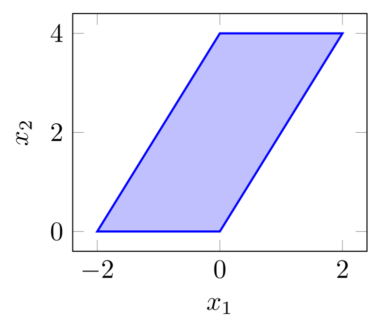
A



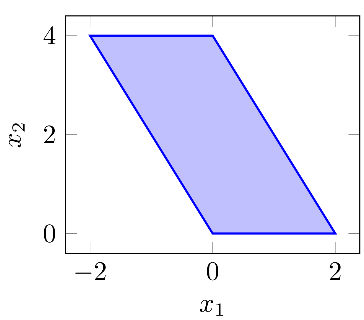
B



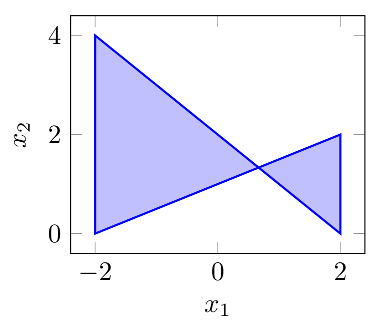
C



D



E



Question 2

Which Linear Program in standard form below is equivalent to the following Linear Program?

$$\begin{array}{rl} \max & x + 2y \\ \text{s.t.} & 2x + y = 4 \\ & 2x \geq y + 5 \\ & x \geq 0 \ . \end{array}$$

A

$$\begin{array}{rl} \max & x + 2y \\ \text{s.t.} & 2x + y \leq 4 \\ & -2x - y \leq -4 \\ & -2x + y \leq -5 \\ & x \geq 0 \ . \end{array}$$

B

$$\begin{array}{rl} \max & x + 2y \\ \text{s.t.} & 2x + y \leq 4 \\ & -2x + y \leq -5 \\ & x, y \geq 0 \ . \end{array}$$

C

$$\begin{array}{rl} \max & x + 2y \\ \text{s.t.} & 2x + y \leq 4 \\ & -2x - y \leq -4 \\ & -2x + y \leq -5 \\ & x, y \geq 0 \ . \end{array}$$

D

$$\begin{array}{rl} \max & x + 2y_1 - 2y_2 \\ \text{s.t.} & 2x + y_1 - y_2 \leq 4 \\ & -2x - y_1 + y_2 \leq -4 \\ & -2x + y_1 - y_2 \leq -5 \\ & x, y_1, y_2 \geq 0 \ . \end{array}$$

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E None of the above

Question 3

Consider the following dictionaries with nonbasic variables x_1, w_1 and basic variables x_2, w_2:

A

$$\begin{array}{lcrrr} \zeta & = & -6 & +1x_1 & -1w_1 \\ \hline x_2 & = & 5 & -2x_1 & +0w_1 \\ w_2 & = & 9 & +3x_1 & -2w_1 \end{array}$$

B

$$\begin{array}{lcrrr} \zeta & = & 6 & +3x_1 & -2w_1 \\ \hline x_2 & = & 4 & -2x_1 & +0w_1 \\ w_2 & = & -1 & -1x_1 & -2w_1 \end{array}$$

C

$$\begin{array}{lcrrr} \zeta & = & -1 & -1x_1 & -2w_1 \\ \hline x_2 & = & 1 & -2x_1 & +0w_1 \\ w_2 & = & 8 & -1x_1 & -2w_1 \end{array}$$

D

$$\begin{array}{lcrrr} \zeta & = & -2 & +1x_1 & +0w_1 \\ \hline x_2 & = & 9 & +2x_1 & +3w_1 \\ w_2 & = & 5 & +1x_1 & -2w_1 \end{array}$$

Which dictionaries are (for each item, zero, one or multiple choices possible):

- Feasible: A B C D
- Final optimal: A B C D
- Final unbounded : A B C D

Question 4

Consider the following feasible dictionary:

$$\begin{array}{lcrrr} \zeta & = & -10 & +3x_1 & -2w_1 \\ \hline x_2 & = & 4 & -6x_1 & +0w_1 \\ w_2 & = & 3 & +3x_1 & -6w_1 \end{array}$$

Which dictionary is obtained after applying one step of pivoting of the simplex algorithm?

A

$$\begin{array}{lcrrr} \zeta & = & -8 & -\frac{1}{2}x_2 & -2w_1 \\ \hline x_1 & = & \frac{2}{3} & -\frac{1}{6}x_2 & +0w_1 \\ w_2 & = & 5 & -\frac{1}{2}x_2 & -6w_1 \end{array}$$

B

$$\begin{array}{lcrrr} \zeta & = & -11 & +2x_1 & -\frac{1}{3}w_2 \\ \hline x_2 & = & 4 & -6x_1 & +0w_2 \\ w_1 & = & \frac{1}{2} & +\frac{1}{2}x_1 & -\frac{1}{6}w_2 \end{array}$$

C

$$\begin{array}{lcrrr} \zeta & = & -6 & -1x_2 & -2w_1 \\ \hline x_1 & = & 4 & -1x_2 & +0w_1 \\ w_2 & = & 5 & -\frac{1}{2}x_2 & -6w_1 \end{array}$$

D

$$\begin{array}{lcrrr} \zeta & = & -13 & +0x_1 & -1w_2 \\ \hline x_2 & = & 4 & -6x_1 & +0w_2 \\ w_1 & = & 3 & +3x_1 & -1w_2 \end{array}$$

Question 5

Consider the following Linear Program:

$$\begin{array}{rl} \max & x_1 + 2x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 4 \\ & -2x_1 - x_2 \leq -4 \\ & -2x_1 + x_2 \leq -5 \\ & x_1, x_2 \geq 0\ . \end{array} \tag{1}$$

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Which of the following Linear Programs is the dual of (1)?

A

$$\begin{array}{ll} \min & y_1 + 2y_2 \\ \text{s.t.} & 2y_1 + y_2 \geq 4 \\ & -2y_1 - y_2 \geq -4 \\ & -2y_1 + y_2 \geq -5 \\ & y_1, y_2 \geq 0\ . \end{array}$$

B

$$\begin{array}{ll} \min & 4y_1 - 5y_2 \\ \text{s.t.} & 2y_1 - 2y_2 \geq 1 \\ & y_1 + y_2 \geq 2 \\ & y_1, y_2 \geq 0\ . \end{array}$$

C

$$\begin{array}{ll} \min & 4y_1 - 4y_2 - 5y_3 \\ \text{s.t.} & 2y_1 - 2y_2 - 2y_3 \geq 1 \\ & y_1 - y_2 + y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0\ . \end{array}$$

D

$$\begin{array}{ll} \min & 4y_1 - 4y_2 - 5y_3 \\ \text{s.t.} & 2y_1 - 2y_2 - 2y_3 = 1 \\ & y_1 - y_2 + y_3 = 2 \\ & y_1, y_2, y_3 \geq 0\ . \end{array}$$

E None of the above

Question 6

Given $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$, consider the following Linear Program in matrix standard form:

$$\begin{array}{ll} \max & c^\top x \\ \text{s.t.} & Ax \leq b \\ & x \geq 0\ . \end{array} \tag{2}$$

(i) Give the expression of the slack variables of (3).

$w =$

(ii) Give the dual of (3).

(iii) Give the expression of the slack variables of the dual of (3).

$z =$

(iv) State the complementary slackness theorem for (3) and its dual.

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Question 7

Given $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, let P denote the following Linear Program:

$$\begin{aligned} \max \quad & c^\top x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \,. \end{aligned} \tag{3}$$

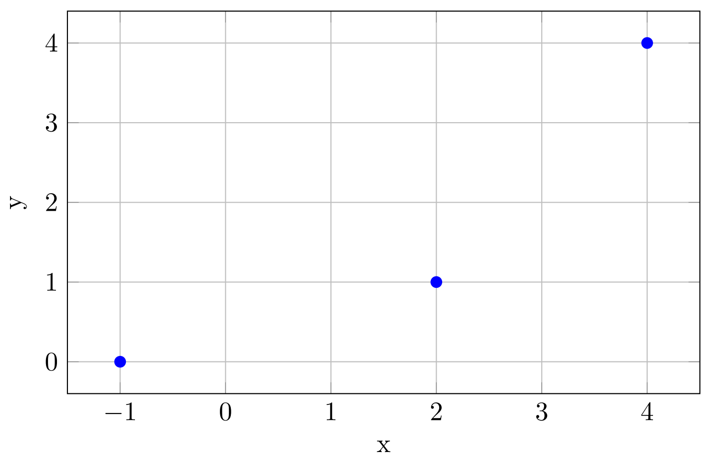
Let D denote its dual.

Which of the following sentences are correct (negative point for incorrect answers):

- If D is feasible, then P is feasible as well. True False
- If P is infeasible, then D is unbounded. True False
- If D is unbounded, then P is infeasible. True False
- If P has a (bounded) optimal solution, then D is feasible. True False
- If D is feasible, then P has a (bounded) optimal solution. True False

Question 8

Consider the following set S of three data points $(\mathrm{x}_i, \mathrm{y}_i)$, $i = 1, 2, 3$ (blue dots):



Which of the following Linear Programs corresponds to the problem of linear L^∞-regression of the points in S, i.e., finding the line $\mathrm{y} = a\mathrm{x} + b$ that minimizes $\max_{i=1,2,3} |a\mathrm{x}_i + b - \mathrm{y}_i|$?

A

$$\begin{aligned} \min \quad & t_1 + t_2 + t_3 \\ \text{s.t.} \quad & a\mathrm{x}_i + b \leq \mathrm{y}_i + t_i \quad \forall i = 1, 2, 3 \\ & a, b, t_1, t_2, t_3 \geq 0 \,. \end{aligned}$$

B

$$\begin{aligned} \min \quad & t_1 + t_2 + t_3 \\ \text{s.t.} \quad & a\mathrm{x}_i + b \leq \mathrm{y}_i + t_i \quad \forall i = 1, 2, 3 \\ & a\mathrm{x}_i + b \geq \mathrm{y}_i - t_i \quad \forall i = 1, 2, 3 \\ & a, b, t_1, t_2, t_3 \geq 0 \,. \end{aligned}$$

C

$$\begin{aligned} \min \quad & t \\ \text{s.t.} \quad & a\mathrm{x}_i + b \leq \mathrm{y}_i + t \quad \forall i = 1, 2, 3 \\ & a, b, t \geq 0 \,. \end{aligned}$$

D

$$\begin{aligned} \min \quad & t \\ \text{s.t.} \quad & a\mathrm{x}_i + b \leq \mathrm{y}_i + t \quad \forall i = 1, 2, 3 \\ & a\mathrm{x}_i + b \geq \mathrm{y}_i - t \quad \forall i = 1, 2, 3 \\ & a, b, t \geq 0 \,. \end{aligned}$$

E None of the above