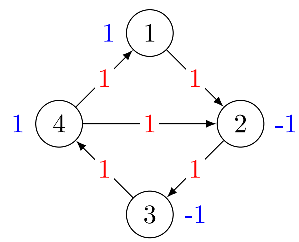
First name, last name:

CSCI 5654, Spring 2023: Spot Exam 2

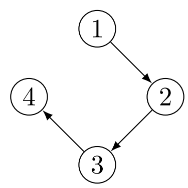
Date: We 5/3/2023

Question 1

Consider the following network:



wherein the numbers besides the nodes are supplies (negative values represent demands) and the numbers on the arcs are unit transportation costs. Consider the following spanning tree:

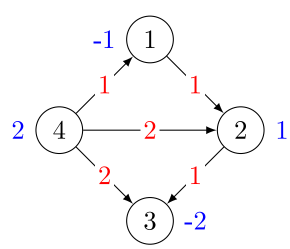


Are the primal and dual solutions associated to this tree:

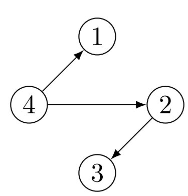
- Primal feasible? Yes No
- Dual feasible? Yes No
- Optimal? Yes No

Question 2

Consider the following network:



wherein the numbers besides the nodes are supplies (negative values represent demands) and the numbers on the arcs are unit transportation costs. Consider the following spanning tree:



Are the primal and dual solutions associated to this tree:

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- Primal feasible? Yes No
- Dual feasible? Yes No
- Optimal? Yes No

Question 3

Consider the following optimization problem involving a disjunctive linear constraint:

$$\begin{array}{rl} \max & t \\ \text{s.t.} & -1 \le x \le 1, \\ & t \le -2x \ \vee \ t \le 3x, \end{array} \tag{1}$$

wherein $x, t \in \mathbb{R}$ are the variables. Fix $M \gg 1$ (e.g., $M = 100$). Which optimization program below is a *Mixed-Integer Linear Program* equivalent to (1)?

A
$$\begin{array}{rl} \max & t \\ \text{s.t.} & -1 \le x \le 1, \\ & t \le -2x + M, \\ & t \le 3x - M \end{array}$$

B
$$\begin{array}{rl} \max & t \\ \text{s.t.} & -1 \le x \le 1, \\ & t \le zx, \\ & z \in \{-2, 3\} \end{array}$$

C
$$\begin{array}{rl} \max & t \\ \text{s.t.} & -1 \le x \le 1, \\ & t \le -2x - Mz, \\ & t \le 3x - M(1-z), \\ & z \in \{0, 1\} \end{array}$$

D
$$\begin{array}{rl} \max & t \\ \text{s.t.} & -1 \le x \le 1, \\ & t \le -2x + Mz, \\ & t \le 3x + M(1-z), \\ & z \in \{0, 1\} \end{array}$$

Question 4

Consider the following nonlinear optimization problem:

$$\begin{array}{rl} \min & t \\ \text{s.t.} & t \ge \sqrt{x_1^2 + 4x_2^2}, \end{array} \tag{2}$$

wherein $x_1, x_2, t \in \mathbb{R}$ are the variables. Which optimization program below is a *Linear & Second-Order Cone Program* equivalent to (2)?

A
$$\begin{array}{rl} \min & t \\ \text{s.t.} & t^2 = x_1^2 + 4x_2^2 \end{array}$$

B
$$\begin{array}{rl} \min & t \\ \text{s.t.} & (t, x_1) \in L_2^1 \\ & (t, 2x_2) \in L_2^1 \end{array}$$

C
$$\begin{array}{rl} \min & t \\ \text{s.t.} & (t, x_1, 4x_2) \in L_2^2 \end{array}$$

D
$$\begin{array}{rl} \min & t \\ \text{s.t.} & (t, 2x_2, x_1) \in L_2^2 \end{array}$$

(Let us remind that $L_2^n = \{(u_0, u_1, \ldots, u_n) \in \mathbb{R}^{n+1} : u_0 \ge \sqrt{u_1^2 + \ldots + u_n^2}\}$.)

Question 5

Given $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and a proper cone $K \subseteq \mathbb{R}^m$, consider the following conic program in standard form:

$$\begin{array}{rl} \min & c^\top x \\ \text{s.t.} & Ax \ge_K b\,. \end{array} \tag{3}$$

(i) Give the definition of the dual cone of K.

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$K^* =$

(ii) Give the dual of (3).

(iii) Let P denote (3) and D denote its dual. Which of the following sentences are correct:

- If D is feasible, then P is feasible as well. True False
- If D is unbounded, then P is infeasible. True False
- If P is strcitly feasible and has an optimal solution, then D is feasible. True False

Question 6

Consider the following Linear Program:

$$\begin{array}{rl} \max & x_2 \\ \text{s.t.} & x_1 + x_2 \leq 3 \\ & x_1 \geq 0, \end{array} \tag{4}$$

and the associated barrier problem:

$$\begin{array}{rl} \max & x_2 + \mu[\log(3 - x_1 - x_2) + \log(x_1)] \\ \text{s.t.} & x_1 + x_2 < 3 \\ & x_1 > 0. \end{array} \tag{5}$$

Which of the following sentences are correct:

- (5) has an optimal solution for all $\mu > 0$.

  True False

- The first-order optimality conditions say that if x^* is an optimal solution of (5) with $\mu > 0$, then

$$\begin{cases} \frac{-\mu}{3-x_1^*-x_2^*} + \frac{\mu}{x_1^*} = 0, \\ 1 + \frac{-\mu}{3-x_1^*-x_2^*} = 0. \end{cases}$$

  True False

(Note that the dual of (4) is given by $\left\{ \begin{array}{rl} \min & 3y_1 \\ \text{s.t.} & y_1 - y_2 = 0 \\ & y_1 = 1 \\ & y_1, y_2 \geq 0 \end{array} \right\}$.)