CSCI 5654, Spring 2023: Assignment 1

Assigned date: Mo 1/23/2023
Due date: Mo 2/6/2023

Instructions: Please upload your HW as a PDF file along with the code that you wrote for running the LP problems. LPs can be solved using any tool of your choice but we recommend Julia, Python (pulp/cvxopt/gurobi), Matlab or GLPK.

Problem 1 (5 points)

Convert the following Linear Program to the *standard form*[1]:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & 2x_1 \geq x_2 - 3 \\ & 2x_1 \leq x_2 + 5 \\ & x_2 \leq 3 \ . \end{aligned}$$

Draw the feasible set and objective function of the above Linear Program (not in standard form) and deduce the optimal solution. Also deduce the optimal solution when the max is replaced by a min (what do you observe?).

Problem 2 (5 points)

Consider the following Linear Program:

$$\begin{aligned} \max \quad & -6x_1 + 9x_2 - 5x_3 - 9x_4 \\ \text{s.t.} \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq 2 \\ & x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \ . \end{aligned}$$

(a) Add *slack variables* to the problem to build an *initial dictionary*. Precise which variables are *basic* (dependent), which are *nonbasic* (independent). (b) Perform one step of the simplex algorithm. Precise which variable is *entering* (becomes basic), which is *leaving* (becomes nonbasic). (c) Find the optimal solution of the problem.

Problem 3 (5 points)

An investment adviser wishes to recommend an ideal investment for her client. After lots of market research, the adviser has compiled a table of investment as expressed in Table 1: it show the investment options and categorizes them based on risk A–D, market segment and whether the investment is in an eco-friendly business. The total investment is $10,000$. The client has specified minimum and maximum investment limits (in term of invested *amount of money*) for each category, as expressed in Table 2.

Setup and solve a Linear Program that maximizes the expected profit while respecting the maximum and minimum investment percentages for each category. Is the problem feasible? Is it bounded? What is the optimal solution?

Note: Upload the code you used to solve the LP as a file `problem3.*`.

Problem 4 (5 points)

Fix $a \in \mathbb{R}^n$ and $b \in \mathbb{R}$. Consider the sets $A = \{x \in \mathbb{R}^n : a^\top x = 0\}$, $B = \{x \in \mathbb{R}^n : a^\top x = b\}$, and $C = \{x \in \mathbb{R}^n : a^\top x \leq b\}$. (a) Show that A is a *linear subspace*, meaning that for all $x, y \in A$ and for all $\lambda, \mu \in \mathbb{R}$, it holds that $\lambda x + \mu y \in A$. (b) Show that B is an *affine subspace*, meaning that for all $x, y \in B$ and for all $\lambda \in \mathbb{R}$, it holds that $\lambda x + (1-\lambda) y \in B$. (c) Show that C is a *convex set*, meaning that for all $x, y \in C$ and for all $\lambda \in [0, 1]$, it holds that $\lambda x + (1-\lambda) y \in C$.

[1] See Vanderbei's book, page 7.

ID	Profit/Unit	Price/Unit	Risk Category	Investment Market	Eco Friendly ?
1	1.451	2.563	A	Tech	Y
2	2.683	4.307	B	Finance	Y
3	5.898	6.422	C	Finance	Y
4	2.102	3.488	A	PetroChem	N
5	5.709	6.581	B	Finance	N
6	4.519	8.993	D	Finance	Y
7	7.176	11.481	C	Finance	Y
8	6.075	11.730	B	Tech	Y
9	5.718	9.270	B	PetroChem	Y
10	7.442	10.160	A	Automobile	Y
11	1.234	1.961	C	Tech	Y
12	4.680	9.300	D	Automobile	N
13	7.229	11.672	A	PetroChem	Y
14	9.589	10.877	B	Automobile	N
15	6.497	12.137	C	Finance	N

Table 1: Investment options and categories.

Risk	Min	Max
A	1500	3500
B	4500	6500
C	1000	3000
D	500	2500

Market	Min	Max
Tech	0	3000
Finance	0	4000
PetroChem	0	5000
Automobile	0	7000

Eco-friendly	Min	Max
Y	2000	10000

Table 2: Investment limits per category.